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EFFECTIVE CONSTITUTIVE RELATIONS FOR LARGE REPETITIVE FRAME-LIKE STRUCTURES

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Abstract—Effective mechanical properties for large repetitive frame-like structures are derived using straight forward combinations of strength of material orthogonal transformation techniques. Once the actual structure is identified symmetry considerations are used in order to identify its independent property constants. The actual values of these constants are constructed according to a building block format which is carried out in the three consecutive steps: (a) All basic planar lattices are identified (b) effective continuum properties are derived for each of these planar basic grids using matrix structural analysis methods and (c) orthogonal transformations are finally used to determine the contribution of each basic set to the overall effective continuum properties of the structure.

1. INTRODUCTION

In recent two papers [1, 2] we introduced a straightforward construction procedure in order to derive continuum equivalence of discrete pin jointed repetitive structures. Broadly speaking we outlined the method as follows: Once the actual structure was specified symmetry considerations were used in order to identify its independent property constants. The actual values of these constants were constructed in accordance with a building block approach consisting of the following three consecutive steps: (a) all sets of parallel members were identified, (b) unidirectional "effective continuum" properties were derived for each of these sets and (c) orthogonal transformations were finally used to determine the contribution of each set to the overall effective continuum properties of the structure. Here the term properties is general and includes mechanical (stiffnesses), thermal (coefficients of thermal expansions) and material densities. The method was then applied to a variety of structures.

In the present paper we extend the analysis of [1, 2] in order to derive the effective properties of rigid-jointed (frame-like) repetitive structures. This differs substantially from the truss-like structures in that we here include the influence of inplane bending rigidities to the structure. The construction procedure will differ in that the rod's unidirectional properties will no longer be adequate to derive the overall properties. The fact that the individual rod in a rigid-jointed array can resist in plane bending dictates that the smallest sub-cell of the structure which will be used for the building block approach will no longer be unidirectional and thus have to be two-dimensional substructures. Here the most identifiable basic two dimensional frame structures are the (0°, 90°) and (0°, ±60°) lay ups. Effective properties for the sub-cells will be constructed using the direct analysis method which is also known by the matrix structural analysis method (see, for example [3–5]). This method, which uses simple and straightforward strength of material techniques, constitutes two-dimensional generalization of the one-dimensional area weighted properties approach of [1, 2]. The derived effective properties for such substructures will then be used in a building block format in order to derive the effective properties of more complicated two and three-dimensional structures. This last step will be done by employing the orthogonal transformation. In summary we thus outline the procedure of constructing effective properties for frame-like repetitive structures as follows. Once the actual structure is identified symmetry considerations are used in order to identify its independent property constants. The actual values of these constants are constructed according to a building block format which is carried out in the three consecutive steps: (a) all basic planar lattices are identified (b) effective continuum properties are derived for each of these planar basic grids. Here a representative repeating cell is isolated and studied by the direct

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method noting that the effect of the joints' rigidity is taken into consideration and (c) orthogonal transformations are finally used to determine the contribution of each basic set to the overall effective continuum properties of the structure.

Since the inclusion of bending rigidities do not influence the thermal expansion of the structure, the thermal expansion properties derived in[1, 2] for the truss are identical to those of corresponding frame. Accordingly in what follows we concentrate on deriving the elastic properties of the frame structure.

2. ORTHOGONAL TRANSFORMATIONS

As was pointed out earlier the actual values of the total structure's effective continuum properties are determined from the individual contribution of each two-dimensional subset. The individual subsets contribution of each two-dimensional subset. The individual subsets contribution is obtained by a three-dimensional coordinate transformation. Before we proceed to describe the transformation, however, we shall first state the relevant stress–strain relations of elastic bodies.

The stress–strain relations for a general linear elastic body are written in the compact form

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl}, \quad i,j,k,l = 1,2,3, \tag{1}$$

where σ_{ij} and ϵ_{kl} are the components of the stesss and strain tensors, respectively and C_{ijkl} are the stiffness tensor of the solid.

For future format references we shall rewrite equation (1) in its expanded form

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1123} & C_{1113} & C_{1112} \\ C_{2211} & C_{2222} & C_{2233} & C_{2223} & C_{2213} & C_{2212} \\ C_{3311} & C_{3322} & C_{3333} & C_{3323} & C_{3313} & C_{3312} \\ C_{2311} & C_{2322} & C_{2333} & C_{2323} & C_{2313} & C_{2312} \\ C_{1311} & C_{1322} & C_{1333} & C_{1323} & C_{1313} & C_{1312} \\ C_{1211} & C_{1222} & C_{1233} & C_{1223} & C_{1213} & C_{1212} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{23} \\ \epsilon_{13} \\ \epsilon_{12} \end{bmatrix}. \tag{2}$$

Since C_{ijkl} is a fourth-order tensor it obeys the transformation[1, 6, 7]

$$C_{ijkl} = C'_{pqrs}\,\beta_{pi}\,\beta_{qj}\,\beta_{rk}\,\beta_{sl} \tag{3}$$

where

$$\beta_{ij} = \frac{\partial x'_i}{\partial x_j} \tag{4}$$

are components of the orthogonal transformation tensor which transforms the unprimed to the primed coordinates. Accordingly, β_{ij} is the cosine of the angle between the x'_i and the x_j axis.

The relation (3) hold equally well for either continuous or discrete structures. The numerical values of the appropriate C_{ijkl} entries will depend, however, upon the specific structure under consideration. Since we are interested in analyzing frame-type structures that are constructed from smaller subsets, it is expected that each subset will contribute to its overall properties.

If a structure has n different subsets then equation (3) can be written for each subset m, $m = 1, 2, \ldots, n$ as

$$(C_{ijkl_m}) = (C'_{pqrs}\,\beta_{pi}\,\beta_{qj}\,\beta_{rk}\,\beta_{sl_m}). \tag{5}$$

Once the direction cosines of each subset are identified the sum over all of these subsets yield the final properties

$$C_{ijkl} = \sum_{m=1}^{n} (C_{ijkl_m}). \tag{6}$$

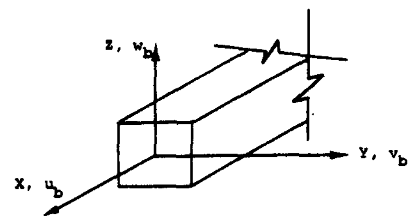
3. BASIC PLANAR GRIDS

We shall use the "direct method" to find the properties of the equivalent continuum of two basic planar grids. This approach is the reverse of that used by McCormick[8], McHenry[9] and Hrennikoff[10], who describe a procedure for modeling problems in plane stress analysis with one-dimensional elements.

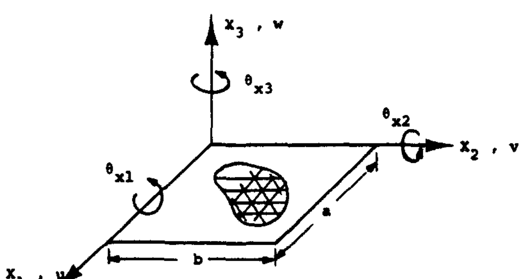
The main idea behind the direct method is to equate the displacements of the nodes of the model to the displacements of the corners of the continuum plate element under the same loading conditions. The sign convention for the displacement and stress resultants used in the present study are shown in sketch 1(a, b).

(a) (0°, 90°) *layup*

We consider a plane network which is formed from a large number of orthogonally intersecting beams rigidly jointed at their intersections as shown in Fig. 1. The beams are



a-The Beam Element in the Single-Layer Grids.



b-Equivalent Plate Sign Convention

Sketch 1. Sign convention for the displacements in the equivalent continuum plate model.

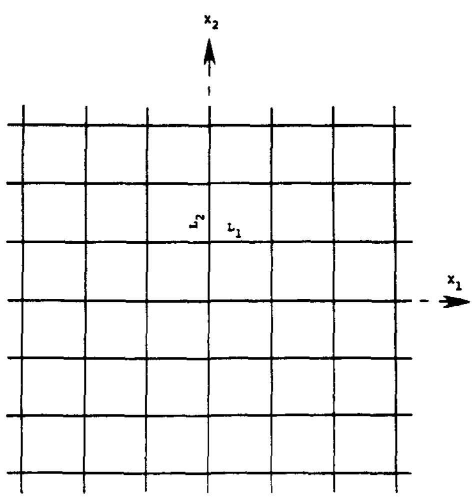


Fig. 1. The (0°, 90°) lattice.

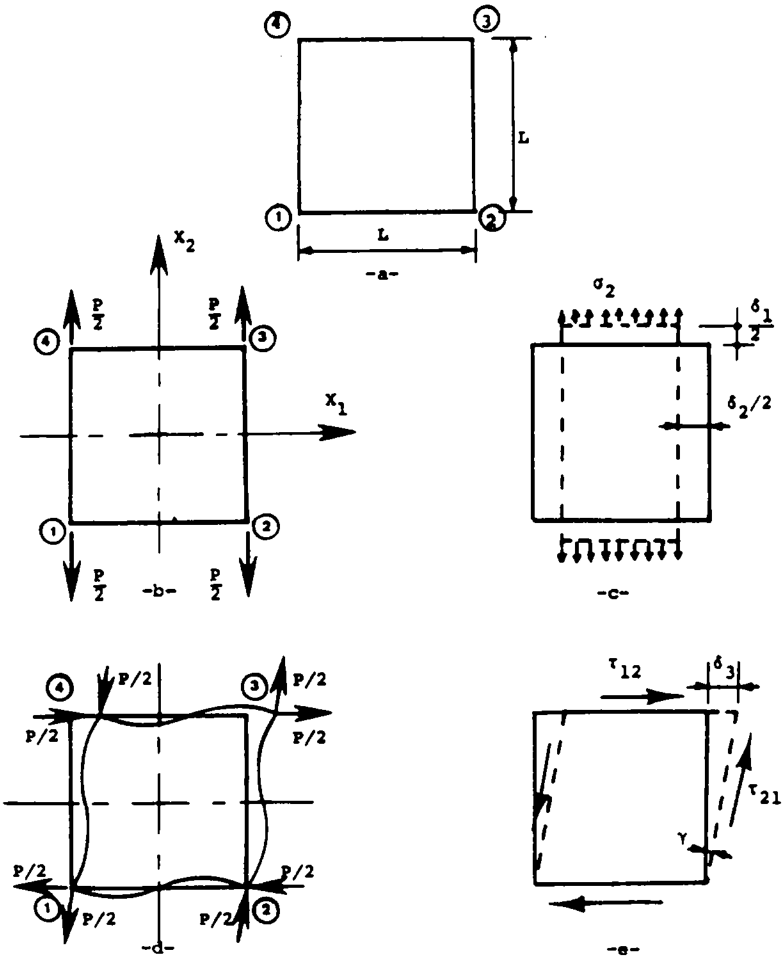
assumed to be identical, each having the length L, the cross-sectional area A, the Youngs modulas E and the moments of innertia I_y and I_z around the Y and Z axis (principal axes), respectively. The deformation of each joint is described by the displacements u, v and w in the X_1, X_2 and X_3 directions, respectively and by the rotations θ_{x1}, θ_{x2} and θ_{x3} around the axis X_1, X_2 and X_3, respectively. Here the rotations are considered to be positive in the counterclockwise direction.

Using symmetry arguements reveal that this model is orthotropic and that a 90° rotation in its plane will not alter its behavior[11]. These conditions reduce its general stress–strain relations (1) to

$$[C_{ijkl}] = \begin{bmatrix} C_{111} & C_{1122} & 0 & 0 & 0 & 0 \\ C_{1122} & C_{1111} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{1313} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{1212} \end{bmatrix} \tag{7}$$

which has the four independent constants C_{1111}, C_{1122}, C_{1313} and C_{1212}.

The actual values of these constants are derived using the direct method of analysis. This consists of isolating the representative repeating cell, Fig. 2(a), loading it at its nodes and equating the displacements of these nodes to the displacements of the edges of the equivalent continuum plate under the same loading conditions. The appropriate loading conditions for



Figs. 2(a–e).

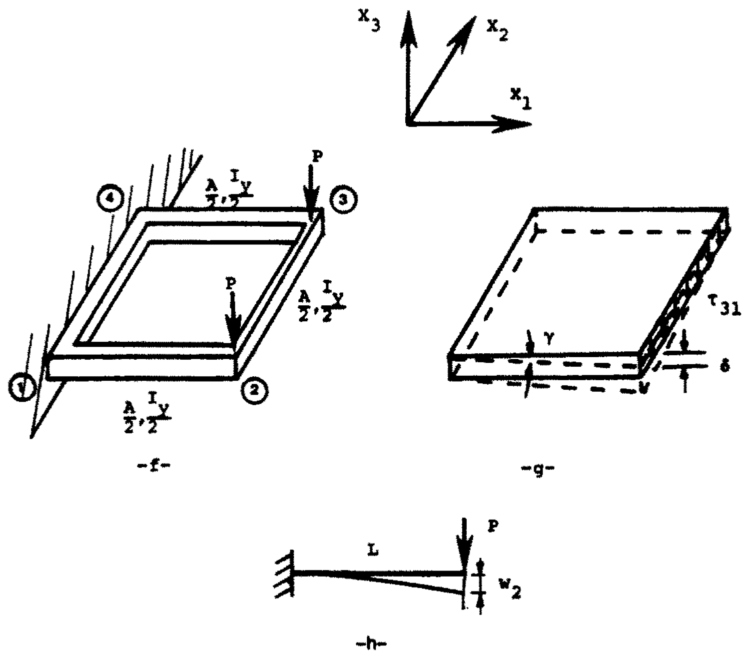


Fig. 2. The loading conditions of the representative repeating cell for the (0°, 90°) layup used to determine the stiffness coefficients of the equivalent continuum.

calculating C_{1111} and C_{1122} are shown in Figs. 2(b, c), those pertaining to calculating C_{1212} are shown in Fig. 2(d, e) and finally those used in calculating C_{1313} are shown in Figs. 2(f–h). In the first and second loading conditions, we are dealing only with the "in-plane" displacements of the lattice; while in the third loading condition we are calculating the "off-plane" displacement.

Since each member is shared by two neighboring cells, its effective cross sectional area and moments of inertia must be half of the corresponding values in the original lattice. Under the present loading conditions, matrix structural methods [3] are utilized to solve for the displacments and rotations of each individual node. Specifically for Fig. 2(b), we obtain

$$u_1 = u_2 = u_3 = u_4 = 0 \tag{8a}$$

$$v_1 = v_2 = -v_3 = -v_4 = -\frac{PL}{2EA} \tag{8b}$$

and from Fig. 2(d), we get

$$u_1 = u_2 = 0, \quad v_1 = v_2 = 0, \quad v_3 = v_4 \tag{9a}$$

$$u_3 = u_4 = \frac{PL^3}{6EI_z} \tag{9b}$$

Similarly, the displacement in Fig. 2(h) is found to be

$$w_2 = w_3 = -PL^3/(3EI_y/2). \tag{10}$$

Figures 2(c, e,f) display the equivalent square continuum element of side length L and thickness h subjected to normal stresses, σ_2, in-plane shearing stresses, τ_{13}, respectively. The displacements of the plate element due to the normal stress σ_2 are

$$\delta_1 = \frac{\sigma_2 L}{E_e}, \quad \delta_2 = \frac{\sigma_2 L \nu_e}{E_e}, \tag{11}$$

and the one due to the in-plane shearing stress τ_{12}, is

$$\delta_3 = \frac{\tau_{12} L}{G_{12}} \tag{12}$$

while the displacement due the off-plane shearing stress τ_{13} is given as

$$\delta = -\frac{\tau_{13} L}{G_{13}} \tag{13}$$

where E_e is the effective modulus of elasticity of the equivalent orthotropic continuum in the X_1 and the X_2 direction, ν_e is the effective Poisson's ratio of the continuum between the X_1 and X_2 direction, G_{12} is the in-plane shear modulus and G_{13} is the off-plane shear modulus. The relations between C_{ijkl} of eqn (7) and E_e, ν_e, G_{12} and G_{13} are

$$E_e = C_{1111}(1-\nu_e^2), \quad \nu_e = \frac{C_{1122}}{C_{1111}} \tag{14a}$$

$$G_{12} = C_{1212}, \quad G_{13} = C_{1313}. \tag{14b}$$

By equating the displacements of the plate element with the corresponding displacements of the representative unit cell while insuring that the total force on the unit cell equals the total force on the plate element for each loading condition yields

$$C_{1111} = \frac{AE}{Lh}, \quad C_{1122} = 0 \tag{15a}$$

$$C_{1212} = \frac{6EI_z}{L^3 h} \tag{15b}$$

$$C_{1313} = \frac{3EI_y}{L^3 h}. \tag{15c}$$

(b) ($0°$, $\pm 60°$ *layups*)

For the (0, $\pm 60°$) layup of Fig. 3, we shall assume that all members are identical and have the same geometrical and material properties L, A, I_y, I_z and E. The isotropic nature of the (0, $\pm 60°$) configuration (see [2, 11]) dictates additional restrictions on the stiffnesses coefficients of

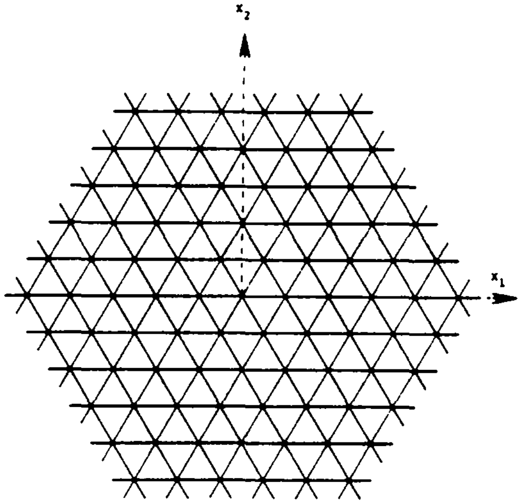


Fig. 3. ($0°$, $\pm 60°$) layup.

the equivalent continuum. The appropriate property matrix is

$$[C_{ijkl}] = \begin{bmatrix} C_{1111} & C_{1122} & 0 & 0 & 0 & 0 \\ C_{1122} & C_{1111} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{1313} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2(C_{1111} - C_{1122}) \end{bmatrix} \quad (16)$$

which has the three independent constants C_{1111}, C_{1122} and C_{1313}. The actual values of these constants are derived using the same method outlined above.

The appropriate loading conditions for calculating C_{1111} and C_{1122} are shown in Figs. 4(a, b), and those used in calculating C_{1313} are shown in Figs. 4(e, f). The representative unit cell for this layup is shown Fig. 4(a). Since the diagonal members are shared by two neighboring cells, their

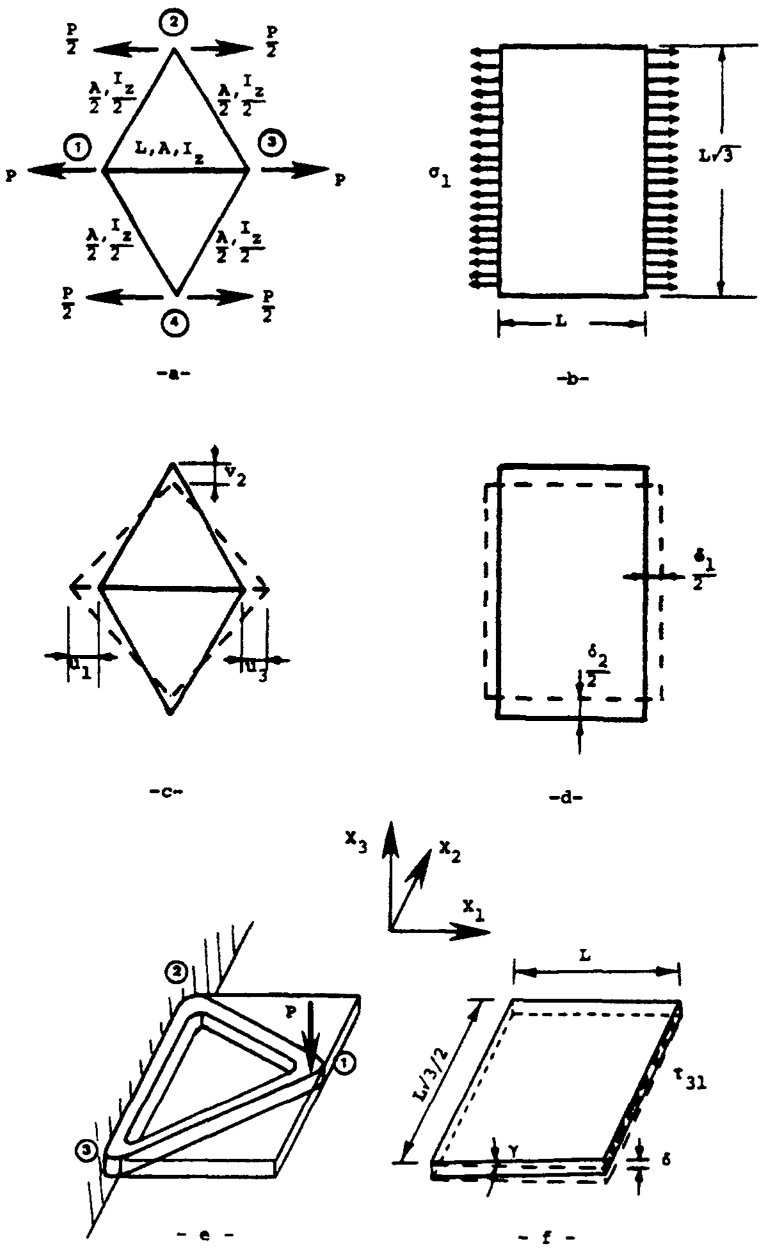


Fig. 4. The loading conditions of the representative repeating cell for the (0°, ±60°) layup used to determine the stiffness coefficients of the equivalent continuum.

effective cross sectional properties are half those of the chord member. With these loading conditions, matrix structural methods are utilized again to solve for the displacements of each individual node. Specifically, from Fig. 4(c) we obtain

$$u_3 = \frac{PL}{2E} \frac{\left(3A + \frac{12I_z}{L^2}\right)}{3A\left(A + \frac{12I_z}{L^2}\right)} \tag{17a}$$

$$\frac{v_2}{u_3} = -\frac{\sqrt{(3A)} - \frac{12\sqrt{(3I_z)}}{L^2}}{3A + \frac{12I_z}{L^2}} \tag{17b}$$

and from Fig. 4(f) we get

$$w_1 = -\frac{PL^3}{3EI_y}. \tag{18}$$

Figure 4(b, f) display the equivalent rectangular continuum element of side dimensions $L \times L\sqrt{3}$ and thickness h, subjected to normal stresses σ_1, and off-plane shearing stresses, τ_{13}, respectively.

The displacements of the plate element due to the normal stress σ_1 are given by

$$\delta_1 = \frac{\sigma_1 L}{E_e}, \quad \delta_2 = -\sqrt{3}\, \nu_e \delta_1 \tag{19}$$

and the displacement due to the off-plane shearing stress τ_{13} is given as

$$\delta = -\frac{\tau_{13} L\sqrt{3}}{2G_{13}}. \tag{20}$$

Using the relations between C_{ijkl} and E_e, ν_e and G_{13} as given in (14), equating the displacements of the plate element with the corresponding displacements of the representative unit cell and insuring that the total force on the unit cell equals the total force on the plate element for each loading condition yields

$$C_{1111} = \frac{3\sqrt{3}\,EA}{4Lh} + \frac{3\sqrt{3}\,EI_z}{L^3 h} \tag{21a}$$

$$C_{1122} = \frac{\sqrt{3}\,EA}{4Lh} - \frac{3\sqrt{3}\,EI_z}{L^3 h} \tag{21b}$$

$$C_{1313} = \frac{3\sqrt{3}\,EI_y}{L^3 h}. \tag{21c}$$

4. APPLICATIONS

In this section we present applications to our construction procedure as outlined in Sections 2 and 3. The models which we shall discuss constitute two-dimensional and three-dimensional beam-like structures, respectively.

(a) *Two-dimensional structures: The* (0°, 90°, ±45°) *layup*

The (0°, 90°, ±45°) grid shown in Fig. 5 is constructed from two basic square grids inclined at an angle of 45° and having the geometrical properties L, E, A, I_y and I_z and $L\sqrt{2}$, E_d, A_d, I_{yd}, I_{zd}, respectively.

The four independent constants for the first (i.e. 0°, 90°) basic square grid with respect to its

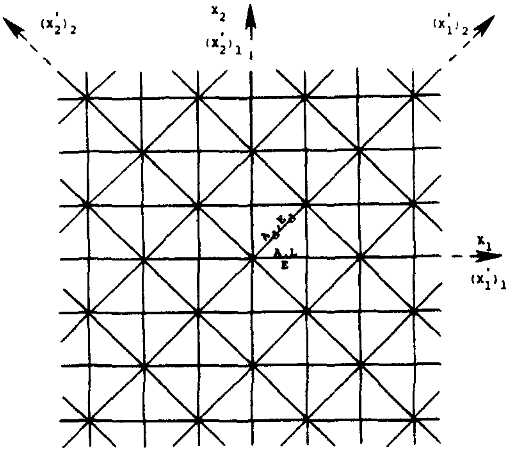


Fig. 5. The (0°, 45°, ±90°) lattice.

local system of axis are given in (15); while those corresponding to the ±45° square grid with respect to its own local system of axis are

$$(C_{1111})_2 = \frac{E_d A_d}{L\sqrt{2}\,h}, \quad (C_{1122})_2 = 0 \tag{22a}$$

$$(C_{1212})_2 = \frac{3E_d I_{zd}}{\sqrt{2}\,L^3 h} \tag{22b}$$

$$(C_{1313})_2 = \frac{3E_d I_{yd}}{2\sqrt{2}\,L^3 h}. \tag{22c}$$

The direction cosines of the local system of axis of the ±45° grid with respect to the fixed coordinate system of axis (X_1, X_2, X_3) are defined according to (4) as

(23)

	X_1	X_2	X_3
$(X_1')_2$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0
$(X_2')_2$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0
$(X_3')_2$	0	0	1

Substituting from (15), (22) and (23) into (5) and summing the results yield the final properties of the 0°, 90°, ±45° layup as

$$C_{1111} = \frac{EA}{Lh} + \frac{E_d A_d}{2\sqrt{2}\,Lh} + \frac{3E_d I_{zd}}{\sqrt{2}\,L^3 h} \tag{24a}$$

$$C_{1122} = \frac{E_d A_d}{2\sqrt{2}\,Lh} - \frac{3E_d I_{zd}}{\sqrt{2}\,L^3 h} \tag{24b}$$

$$C_{1212} = \frac{E_d A_d}{2\sqrt{2}\,Lh} + \frac{6EI_z}{L^3 h} \tag{24c}$$

$$C_{1313} = \frac{3EI_y}{L^3 h} + \frac{3E_d I_{yd}}{2\sqrt{2}\,L^3 h} \tag{24d}$$

(b) *Three-dimensional structures*: (*octetruss structures*)

The smallest generating (repeating) unit cell of the octetruss structure is shown in Fig. 6. It is a diamond-like element with each of its sides having the length L and being shared by two neighboring cells. The octetruss structure is shown in Fig. 7 with respect to the coordinate

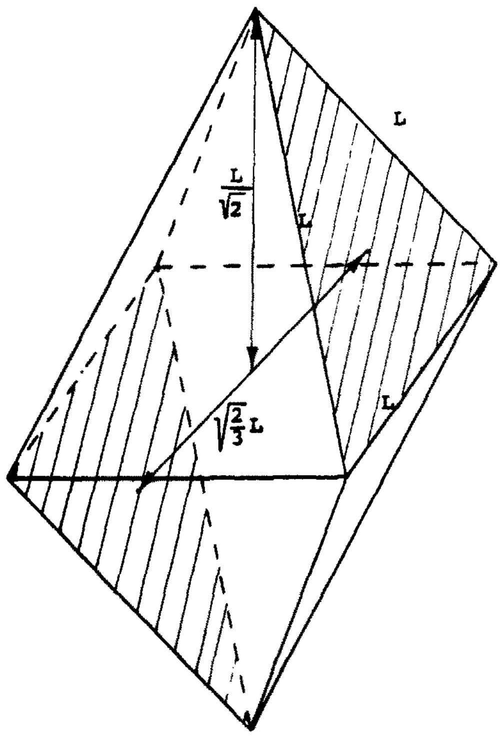


Fig. 6. Smallest repeating element of the octetruss structure.

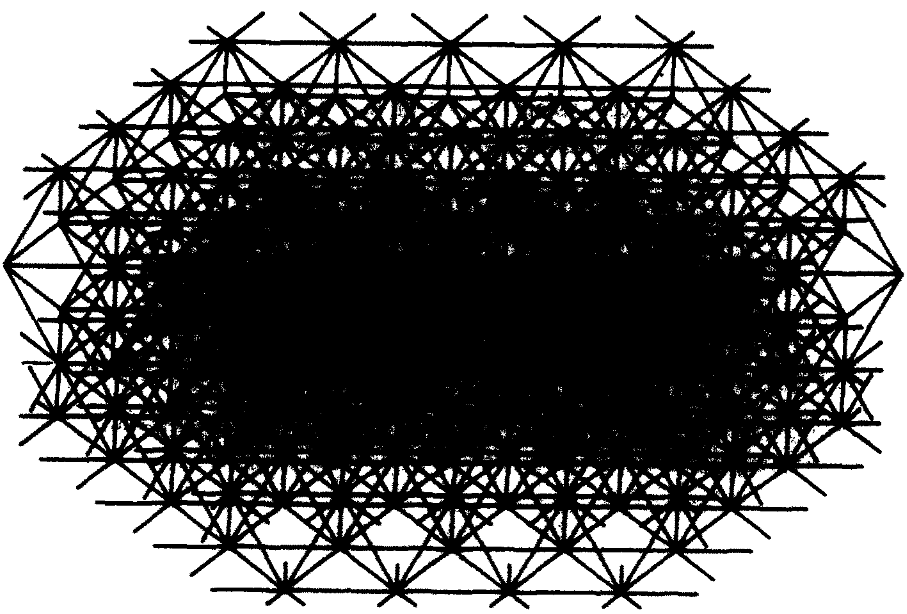


Fig. 7. Three-dimensional octetruss structure viewed with respect to the coordinate system of Fig. 8.

system arrangement shown in Fig. 8. For further details of the geometric characteristics of this kind of structure the reader is referred to [12]. In the present analysis, the octetruss structure is considered to be composed of "beam elements." Examination of this structure reveals that it can be constructed from the superposition of different planes. Specifically, it can be constructed from the three repeating sets of (0°, 90°) basic planar grids having different orientation in space, as shown in Fig. 9. The stiffness coefficients for each of the (0°, 90°) basic grid with respect to its local system of axis are given in (15) where h now stands for the distance between the

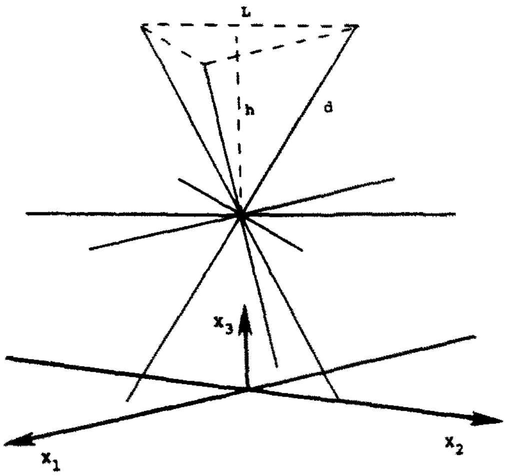


Fig. 8. Direction cosines of the octetruss.

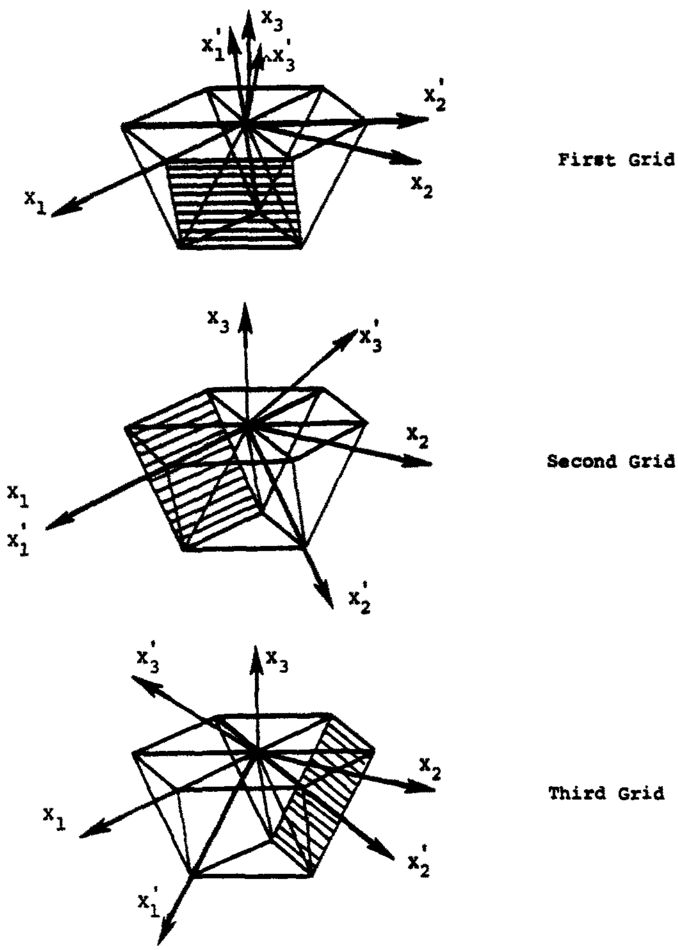


Fig. 9. The octetruss structure constructed from three basic planar (0°, 90°) grids viewed in the coordinate system of Fig. 8.

parallel (0°, 90°) layers; its value is thus given by

$$h = \frac{L}{\sqrt{2}}. \tag{25}$$

The direction cosines of the local system of axis of the three basic (0°, 90°) planes with respect to the global system of the axis of Fig. 8 are defined according to eqn (4) as (β_{ijm}), $m = 1, 2, 3$ by

$$(\beta_{ij})_1 = \begin{bmatrix} 1/2 & 1/2\sqrt{3} & \sqrt{(2/3)} \\ -1/2 & \sqrt{(3/2)} & 0 \\ -1/\sqrt{2} & -\sqrt{(1/6)} & \sqrt{(1/3)} \end{bmatrix} \tag{26a}$$

$$(\beta_{ij})_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{(1/3)} & -\sqrt{(2/3)} \\ 0 & \sqrt{(2/3)} & \sqrt{1/3)} \end{bmatrix} \tag{26b}$$

$$(\beta_{ij})_3 = \begin{bmatrix} 1/2 & -(1/2\sqrt{3}) & -\sqrt{2/3} \\ 1/2 & (\sqrt{3/2}) & 0 \\ \sqrt{(1/2)} & -\sqrt{1/6} & \sqrt{(1/3)} \end{bmatrix}. \tag{26c}$$

Substituting from (26) into (5), using (15) and summing according to (6) yields the final properties of the octetruss structure with respect to coordinates of Fig. 8 as

$$[C_{ijkl}] = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1123} & 0 & 0 \\ C_{1122} & C_{1111} & C_{1133} & -C_{1123} & 0 & 0 \\ C_{1133} & C_{1133} & C_{3333} & 0 & 0 & 0 \\ C_{1123} & -C_{1123} & 0 & C_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{2323} & C_{1123} \\ 0 & 0 & 0 & 0 & C_{1123} & C_{1212} \end{bmatrix} \tag{27}$$

where

$$C_{1111} = \frac{5\sqrt{2}}{4}\frac{EA}{L^2} + 6\sqrt{2}\frac{EI_y}{L^4} + 3\sqrt{2}\frac{EI_z}{L^4} \tag{28a}$$

$$C_{1122} = \frac{5\sqrt{2}}{12}\frac{EA}{L^2} - \frac{2\sqrt{2}\,EI_y}{L^4} - 3\sqrt{2}\frac{EI_z}{L^4} \tag{28b}$$

$$C_{1133} = \frac{\sqrt{2}}{3}\frac{EA}{L^2} - 4\sqrt{2}\frac{EI_y}{L^4} \tag{28c}$$

$$C_{1123} = \frac{1}{6}\frac{EA}{L^2} + 4\frac{EI_y}{L^4} - 6\frac{EI_z}{L^4} \tag{28d}$$

$$C_{3333} = \frac{4\sqrt{2}}{3}\frac{EA}{L^2} + \frac{8\sqrt{2}\,EI_y}{L^4} \tag{28e}$$

$$C_{2323} = \frac{\sqrt{2}}{3}\frac{EA}{L^2} + 2\sqrt{2}\frac{EI_y}{L^4} + 6\sqrt{2}\frac{EI_z}{L^4} \tag{28f}$$

$$C_{1212} = \frac{5\sqrt{2}}{12}\frac{EA}{L^2} + 4\sqrt{2}\frac{EI_y}{L^4} + 3\sqrt{2}\frac{EI_z}{L^4} \tag{28g}$$

Notice that (27) constitutes a modification of our previously reported result in [1] which are reflected in the appearance of the bending rigidties of the members. Notice also that there is no change in the number of the independent constants which can also be deduced from symmetry [1, 2]. Examination of the results (28) indicates that $C_{1212} = (C_{1111} - C_{1122})/2$ and hence the octetruss is transversely isotropic, as is expected.

Remark

By reexamining Fig. 7 we can see that the same structure can also be constructed from four member will be shared by two different basic grids. Since I_y and I_z are the moments of inertia of the cross section of the beam around two principal axes and since each beam is shared by two different basic grids, we must have two sets of principal axis for each cross section; this can only sense for circular cross-sections. Thus, constructing the properties of the octetruss from those pertaining to four (0°, ±60°) layups is restrictive in that only beams with circular cross-section can be treated. This was actually done in[12] and its results were found identical to (27) and (28) when the later are also specialized to $I_y = I_z$.

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